The Second International Mathematics Study

AN ANALYSIS OF SELECTED ACHIEVEMENT DATA MATHEMATICS 8 AND ALGEBRA 12

A REPORT TO
THE MINISTRY OF EDUCATION
PROVINCE OF BRITISH COLUMBIA

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THE SECOND INTERNATIONAL MATHEMATICS STUDY

An Analysis of Selected Achievement Data from the Second International Mathematics Study

by

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Table of Contents

PREFACE v

1. Background to the International Study 1

2. Population A Results 15

3. Population B Results 39

4. Conclusion 57

PREFACE

The Second International Mathematics Study was carried out in British Columbia during the 1980-81 school year, and in most other places during the 1981-82 academic year. A preliminary analysis of the British Columbia results was published by the Ministry of Education in the spring of 1982. That report seems to have had little impact, perhaps because it included no comparative data regarding student outcomes.

The international data sets are now available, and the three international reports are in preparation. This brief report is intended to provide persons in British Columbia who are interested in the teaching and learning of mathematics with results from several other jurisdictions; notably, Ontario and the United States.

In a very real sense, this report has been in preparation for three years and a large number of people have contributed ideas and suggestions over that period of time. I am indebted to those of my colleagues who have contributed in this way, and I wish publicly to acknowledge that debt. I also wish to thank Irene Sample who did most of the data analyses used in the preparation of this report, and Sandy Luedtke who did the word-processing in such a professional manner.

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CHAPTER 1

Background to the International Study

During the 1980-81 school year, several thousand British Columbia mathematics students in Grade 8 and Grade 12 and their teachers took part in the Second International Mathematics Study. All of the countries listed in Figure 1 participated in one or more of the phases of the study, and Canada was represented through the participation of British Columbia and Ontario. In most places, the data for the study was collected during the 1981-82 school year, but to date only a few countries have published national reports. Similarly, the international reports have yet to appear; however, they are nearing completion and should be published in 1986.

The Second International Mathematics Study was sponsored by the International Association for the Evaluation of Educational Achievement (IEA). IEA is an association of educational research organizations and ministries of education whose primary goals are to conduct research on an international level and to assist member-states in undertaking cooperative research projects. IEA has conducted international surveys in the past, including the first mathematics study (Husén, 1967), and the Six Subject Survey (Peaker, 1975; Walker, 1976). Twelve countries participated in the first mathematics survey, but Canada was not among them.

Australia	Israel
Belgium (Flemish)	Ivory Coast
Belgium (French)	Japan
Canada (British Columbia)	Luxembourg
Canada (Ontario)	Netherlands
Chile	New Zealand
England	Nigeria
Finland	Scotland
France	Swaziland
Hong Kong	Sweden
Hungary	Thailand
Irish Republic	United States

Figure 1: Countries participating in the Second International Mathematics Study.

In British Columbia the Second International Mathematics Study was sponsored by the Learning Assessment Branch of the Ministry of Education, and a preliminary report of the British Columbia results was published in the spring of 1982

(Robitaille, O'Shea, Dirks, 1982). The goal of the present report is to examine some of the British Columbia results in the light of those from several other jurisdictions which have educational systems and traditions similar to ours, or in which there is a good deal of local interest. These include Ontario, the United States, New Zealand, Sweden, Japan and England and Wales.

The Second International Mathematics Study was a complex and comprehensive project, and it would not be possible in a report as brief as this to deal with all of its aspects in any meaningful way. The two areas which are considered in some detail in this report are a comparison of the mathematics curriculum offered in British Columbia with those offered elsewhere, and an analysis of students' performance on the achievement tests developed for use in the study. The remainder of this first chapter contains a description of the structure of the Second International Mathematics Study and of its conceptual framework. Chapters 2 and 3 deal with the achievement results and comparisons of curricula at the two age-grade levels at which the study was conducted.

In general terms, these two levels were junior and senior secondary. In British Columbia, this involved students enrolled in Mathematics 8 and Algebra 12, respectively. As will be seen later, there were significant differences internationally in the way these two groups of students were defined; particularly the senior one.

Framework for the Study

The overall goal of the Second International Mathematics Study was to produce an international portrait of the teaching and learning of mathematics; and, to this end, a variety of instruments were developed to compare and contrast mathematics curricula, instructional practices, and student outcomes, affective as well as cognitive, in an international context. These three perspectives--curricular, pedagogical, and outcome-oriented--correspond to the three versions of the curriculum identified in the framework for the study: the *intended curriculum*, the *implemented curriculum*, and the *attained curriculum*. That framework is displayed in Figure 2.

The intended curriculum is the mathematics curriculum as defined by the Ministry of Education, and as codified in official documents such as curriculum guides and circulars. The implemented curriculum is the curriculum as taught by teachers in their classrooms to particular groups of students. The attained curriculum is the curriculum as learned by the students and as demonstrated by cognitive and affective outcomes.

It has been argued elsewhere (Robitaille & Dirks, 1982) that significant differences exist among these three operationalizations of the mathematics curriculum, not only in British Columbia, but wherever mathematics is taught. Teachers do not follow curriculum guides slavishly. They make decisions every day about how best to meet the mathematical needs of their students, by selecting items of content and using pedagogical practices which they consider most appropriate. They use curriculum guides, but only as one of a number of sources of information and assistance.

Students do not learn all of the material presented to them; in fact, they do not necessarily learn what their teachers say they intended to teach. Thus, a teacher's view of the nature, role, and importance of mathematics might not be shared by his students. They might view the subject as it affects their career choices and chances for academic success rather than as an object worthy of interest for its own sake.

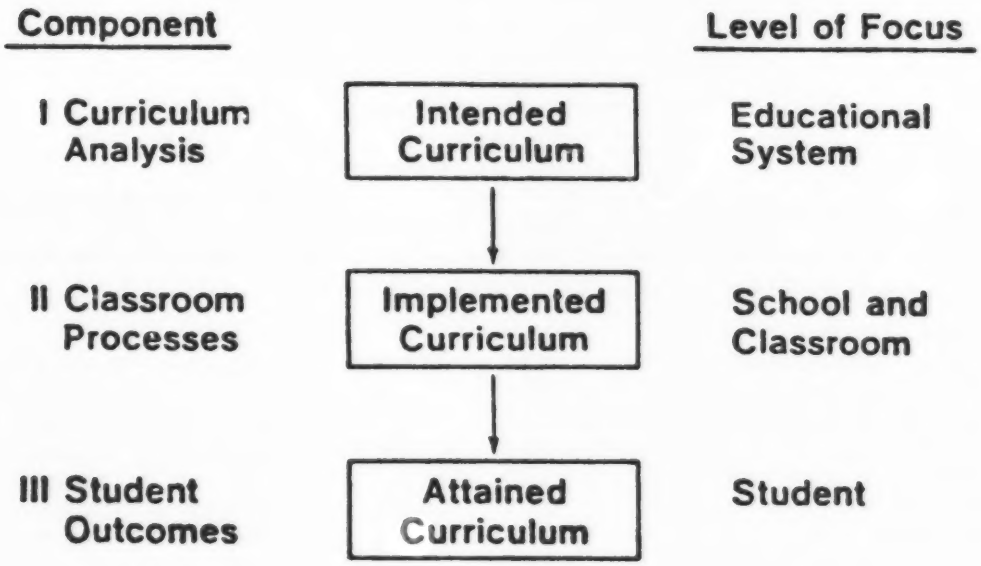

Figure 2: Framework for the international study.

Structure of the Study

The study thus had a three-fold objective: to investigate the nature of the mathematics curriculum being taught in the countries participating in the study, to compare the ways in which mathematics was taught, and to assess the effects of that teaching through an evaluation of students' learning of and attitudes toward mathematics.

Two age-grade levels were selected for the investigation and they are referred to as Populations A and B. Population A was defined to include all students in the grade in which the modal number of students attained the age of 13 by the middle of the school year. In most countries virtually the entire age cohort is still in school at this age level, and almost all students are still studying mathematics. In North America, Population A corresponds to Grade 8 and is usually part of the secondary school system, as it is in most countries. A notable exception occurs in Ontario where Grade 8 is normally the last year of elementary school. Some descriptive data about the structure of Population A are displayed in Table 1.

Table 1
Structure of Population A

Participant	% of Age Cohort in School	Time for Math/Year	
		Hours	Percent
Canada (B.C.)	100	128	14
Canada (Ontario)	100	140	12
U.S.A.	100	146	15
England & Wales	-[a]	117	14
France	100	128	13
Japan	100	99	12
New Zealand	100	132	13
Scotland	100	148	17
Sweden	100	90	11

[a] Data not available.

Population B was defined to include students in the normally accepted terminal grade of the secondary school system who were studying mathematics as a substantial part of an academic program. In other words, Population B included students whose mathematical training was intended to prepare

them to study mathematics at the university level. Whereas Population A included virtually the entire age cohort in most countries, Population B included only those who were specializing to some extent in the study of mathematics and science. In British Columbia, Population B was defined to be all students enrolled in Algebra 12.

The age-grade composition of Population B varies considerably from place to place, as does the proportion of students included. In England, for example, Population B consisted of students in the second year of sixth form studying at least one mathematics course for the General Certificate of Education (G.C.E.) at the Advanced (A-level) or Scholarship level.

In Ontario, the Population B sample for the international study was selected from among those students in Grade 13 who were taking at least two mathematics courses, including Calculus. Fortunately, for the purposes of this report, Ontario also drew a sample of Grade 12 classes, thereby providing a group of students comparable in age and academic preparation to Algebra 12 students in British Columbia and pre-calculus students in the United States. Some comparative information on the structure of Population B is provided in Table 2.

Table 2
Structure of Population B

Participant	% of Age Cohort in School	Pop.B/Grade Cohort %	Pop.B/Age Cohort %	Hours/ Year
Canada (B.C.)	82	38	30	115
Canada (Ont.12)	74	41	30	-
Canada (Ont.13)	33	55	19	110
U.S.A.	72	15	12	153
England & Wales	17	35	6	165
Japan	92	13	12	148
New Zealand	17	67	11	140
Sweden	24	50	12	120

The information contained in the table can be clarified by means of an example. The entries in the first three columns indicate that, in British Columbia, 82% of the age cohort is still in school in the Population B year, Grade 12; 38% of the students in the Grade 12 year are enrolled in

Algebra 12; and Population B represents 30% of the age cohort. Finally, on the average, 115 hours of class time are allotted to the Algebra 12 course.

An examination of these results reveals the existence of substantial differences between countries at the Population B level. Japan has the lowest drop-out rate with over 90% of the age cohort remaining in school until the end of secondary school. British Columbia is second at 82%. At the other extreme New Zealand and England and Wales retain only 17% of the age cohort until the end of secondary school. British Columbia and Ontario offer pre-university mathematics to 30% of their Grade 12 age cohort, the only two jurisdictions shown where this proportion exceeds 20%.

It is clear that, in other countries, a much higher degree of specialization has taken place and students have either opted out or been counselled out of the college-preparatory mathematics program. This is not to say that such students are not taking mathematics; indeed they may well be. For example, the 19% shown for Ontario's Grade 13 Population B represents only those students taking two or three of the Grade 13 courses. Presumably, there are many other students who take only one course and would, therefore, not be completely excluded from the study of mathematics.

The existence of such substantial differences in the makeup of Population B internationally makes it very difficult to draw achievement comparisons between groups. Consider British Columbia and New Zealand for example. It is clear that these are very different groups of students. If one assumes that, in general, the most academically talented students will be included in Population B, then we are comparing approximately the top 10% of New Zealand students to the top third of their counterparts in British Columbia. It is also true that a curriculum designed for the top 10%, or some other relatively small proportion, will be vastly different from one designed for one-third of the population. Thus, in many countries outside North America, it is customary for students to take a full course in Calculus during the Population B year.

Rationale for British Columbia Participation

Participation in the Second International Mathematics Study involved a significant contribution of time and effort on the part of over 200 teachers of mathematics in British Columbia and their students. In addition, a great deal of time, effort, and money were devoted to the preparation, printing, and distribution of the more than 30 data-collection instruments used in the study and to their subsequent

analysis. However, the benefits to be derived from such participation were deemed to be of sufficient importance to justify the costs of participating in the project.

Foremost among the reasons for British Columbia participation was the forthcoming revision of the mathematics curriculum in the province. The information to be obtained from the curriculum analysis phase of the project on an international level, as well as the local results published in this report, was deemed to be of considerable importance to the process of curriculum revision. This type of information makes it possible to examine the British Columbia curriculum in an international context, and provides the curriculum revision committees with data with which to make decisions on the future direction of the mathematics curriculum in this province. Finally, the nature of the information to be collected in such a study is extremely valuable, and its value is enhanced by participating in the study rather than merely reading the results obtained in other countries.

The British Columbia Sample

The structure of the international study afforded countries a fairly wide degree of latitude in matters affecting the size and composition of the sample of classes and teachers selected, in the period of time during which data collection instruments were administered, and in the numbers of instruments that were actually used in each country. Insofar as the sampling was concerned, a major source of between-country variation in the size and composition of the sample selected was the level of analysis in which each country was interested.

Since the major objective for British Columbia participation was to obtain data describing the teaching and learning of mathematics at the provincial level, and not to draw between- or within-school comparisons, a sample size of approximately 100 Grade 8 and 100 Grade 12 classes was established. The following description is a summary of the British Columbia sampling plan which was approved by the IEA Sampling Referee[1] for the Second International Mathematics Study.

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[1] The International Sampling Committee for the Second International Mathematics Study was chaired by Dr. Malcolm Rosier of the Australian Council for Educational Research. The British Columbia sampling plan was originally drawn up by Todd Rogers of the University of British Columbia and later revised by Tom O'Shea of Simon Fraser University and Mary Cooper of British Columbia Research.

Background

For the British Columbia sample, Population A was defined to include all students enrolled in regular Grade 8 classes in the public schools of the province as of September 1980. Excluded by this definition were the approximately 5% of the age cohort enrolled in independent schools as well as those following programs where the level of material covered was significantly below that prescribed for the Mathematics 8 course. Population B was defined to include all students enrolled in Algebra 12 classes in the public schools as of September 1980. This excluded classes which began the course in the second semester.

In order to achieve a sample size of approximately 100 Grade 8 and 100 Algebra 12 classes stratified according to geographic zone of the province and by school size, initial samples of 125 classes at each level were drawn. In most cases this resulted in the selection of no more than one class per school. Of the 125 classes, 105 were selected for initial contact and the remainder reserved to be used as needed. In the spring of 1980, letters were sent from the Ministry of Education to all of the principals of the schools selected, soliciting their cooperation in the study and asking them to select a Mathematics 8 or Algebra 12 teacher or teachers at random from among the teachers available. In cases where it was not possible to make a random selection, the principals were asked to exercise their best judgment about which teacher or teachers to select.

Return Rates

After the sample had been drawn and the materials for the study distributed to the schools, a problem developed in the Grade 8 sample which resulted in the loss of 12 of the 105 Mathematics 8 classes in the sample. In the schools in which these 12 classes were located, all of the Mathematics 8 classes established at the beginning of the school year were disbanded at the end of the first semester, and new classes were set up for the second half of the Mathematics 8 course. Since the students did not stay together but were distributed among several classes and teachers, it was not possible to include them in the study. Unfortunately, this problem was not identified until it was too late in the course of the study to obtain replacements for those classes.

A summary of the return rates for both the Mathematics 8 and Algebra 12 samples is presented in Table 3.

Table 3
Summary of Returns

	Mathematics 8 (Population A)	Algebra 12 (Population B)
Number of classes selected	105	105
Complete Returns	79 (75%)	94 (90%)
Partial Returns	11 (11%)	3 (3%)
Non-participants	15 (14%)	8 (8%)

There were several reasons for the incomplete returns as well as for the non-participation of certain classes. In addition to the organizational problem already mentioned, labour disputes by support staff in a number of school districts, a strike by postal workers, teacher illness, and teachers transferring schools or school districts accounted for almost all of the remaining problems. In only a very few cases did teachers decide not to complete one or more of the instruments because of an unwillingness to cooperate further with the project.

Representativeness of the Achieved Samples

An analysis of the sample of classes and teachers from which complete returns had been obtained was conducted in order to ascertain how representative the achieved sample was of the design sample, and hence of the province as a whole. The samples had been drawn initially using geographic zone of the province and school size as stratification variables.

Population A. At the Population A level, although a substantial number of classes had been lost, those losses were spread across the six geographic zones of the province in such a way that, in the achieved sample, each zone was appropriately represented. The data relative to this matter are presented in Table 4.

Data were not available to enable comparisons to be made directly between the sample of Grade 8 teachers of mathematics who participated in the international study and the population of Mathematics 8 teachers. However, data on a representative sample of teachers of mathematics from Grades 8 to 12 were available from the 1981 Mathematics Assessment, and these were used as a basis for comparison.

Teachers who were selected to participate in the international study at the Population A level were more likely to be mathematics specialists and more experienced than the general population of teachers of secondary mathematics. Teachers of Mathematics 8 who were in the IEA sample had an average of 14 years of teaching experience, compared to 9 years for the population of teachers of secondary mathematics. There were also indications that the IEA teachers spent a greater proportion of their teaching time conducting mathematics classes than did the population of teachers of secondary mathematics. This latter finding is not surprising since principals were asked to select a teacher of mathematics to participate in the study. It is unlikely, in such circumstances, that they would have selected a teacher who taught several subject areas, or whose specialization was in a field other than mathematics.

Table 4
Population A
Geographical Distribution of the Sample

Zone Number	Design Sample		Achieved Sample	
	No. of Classes	Percent	No. of Classes	Percent
1	16	15	13	16
2	39	37	25	32
3	10	10	10	13
4	19	18	13	16
5	7	7	7	9
6	13	13	11	14
TOTAL	104	100	79	100

At the student level, results show that the Mathematics 8 sample was composed of almost equal numbers of boys and girls, the same as was reported for this grade level in the 1981 Mathematics Assessment. The median age of students in the IEA classes was 13.8 years, the same as in the Grade 8 population as a whole. Also, 87% of the IEA teachers categorized the Mathematics 8 curriculum as they presented it to their IEA classes as "typical" rather than as "remedial" or "enriched," and 65% rated their IEA classes as "about the same" in mathematics ability as other Grade 8 mathematics classes. About 30% rated their classes as being of higher ability.

Population B. Rates of complete returns at the Population B level were extremely high. Of the 105 classes in the sample, complete returns were obtained from 90%. As is shown in Table 5, the returns were well distributed among the six geographic zones of the province.

Table 5
Population B
Geographic Distribution of the Sample

Zone Number	Design Sample		Achieved Sample	
	No. of Classes	Percent	No. of Classes	Percent
1	15	14	15	16
2	46	44	42	45
3	8	8	6	6
4	19	18	15	16
5	7	7	7	7
6	10	10	9	10
TOTAL	105	101	94	100

Teachers of IEA classes at the Grade 12 level were more experienced than the population of teachers of mathematics at the senior secondary level: that is, Grades 11 and 12. IEA teachers had an average of 16 years experience compared to 14 for the senior secondary population as a whole. In addition, the IEA teachers were more likely to be specialists in mathematics than the population of teachers of secondary mathematics as a whole. They spent an average of 73% of their teaching time in mathematics classes, whereas the population as a whole reported a figure of 54%. This means, in fact, that the IEA teachers taught one more mathematics class per day than the average.

An analysis of the sample of students in IEA classes at the Grade 12 level by gender shows that 60% were male and 40%, female. This compares with figures of 58% male and 42% female reported in the 1981 Assessment. The median age of all Grade 12 students in the province was 17.8 years in the spring of 1981, while the median age in IEA classes was 17.7. Insofar as the subject matter taught in the IEA classes was concerned, 77% of the teachers in the sample described the Algebra 12 course they taught as "typical" rather than "remedial" or "enriched," and 43% rated their IEA classes as being about equal in ability to other Grade 12 mathematics classes in the

school, a designation that is, unfortunately, more than a little ambiguous since it could include students from several courses other than the Algebra 12 course. Of the remaining teachers, 12% said that theirs was the only Grade 12 mathematics class in the school, and 34% stated that their IEA class was of higher ability than other Grade 12 mathematics classes.

Summary. On the whole, it appears that, at both levels, the geographic distribution of classes participating in the international study is sufficiently close to that of the design sample to be representative of the province as a whole. Moreover, the gender and age statistics for the students in the two samples compare favorably with those for the populations. In the case of the teachers selected for participation in the study, there are some significant differences between them and the population of teachers of mathematics. In particular, the IEA teachers are more experienced and more likely to have specialized in the teaching of mathematics than their colleagues. Put another way, few teachers of subjects other than mathematics who teach one or more mathematics classes in order to complete their teaching schedules were selected for participation; neither were inexperienced teachers. This method of selecting participating teachers seems reasonable, given the objectives of the study.

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CHAPTER 2

Population A Results

The Grade 8 mathematics program in British Columbia is typical of many such programs in North America. The program, as described in the curriculum guide (Curriculum Development Branch, 1978), is divided among eight strands and includes topics from arithmetic, geometry, algebra, measurement, and graphing. According to the curriculum guide, all students in Grade 8 are supposed to follow the same mathematics program; in fact, most schools and teachers report that students are streamed into different programs at this level on the basis of ability.

The content of the mathematics curriculum at the Population A level, and the question of whether or not students of this age are grouped by ability are, of course, related. In Ontario, Population A students are in the last year of elementary school and little streaming takes place: all students follow the same course. In the United States, several distinct courses exist at the Grade 8 level. The best students in most schools, perhaps as much as 20% of the grade cohort, take a full year of algebra; thereby gaining a year of mathematics on their peers. A subset of this group will be eligible to take Advanced Placement courses in Grade 12; and a substantial number, about 5% of the cohort, do take the Advanced Placement Calculus course. The remainder of Grade 8 students in the United States either take a course very similar to the British Columbia Mathematics 8 course, or some form of remedial or general mathematics course.

There is no agreement internationally concerning the appropriateness of streaming students at this level. Among those countries which report that all Population A students take the same course are France, Japan, and New Zealand. On the other hand, data from England and Wales, Sweden, and the United States indicate that streaming by ability is the general practice. In British Columbia, although the Curriculum Guide sets out only one program to be followed by all students, the majority of schools report streaming students on the basis of ability into a variety of enriched and so-called "modified" versions of the Mathematics 8 course.

Given the diversity of educational policies, pedagogical practices, and sociological factors which exist across the more than twenty countries which participated in the Second International Mathematics Study, it is very difficult to make comparisons of students' achievement. It is also true that, to a large extent, it is not possible to quantify the impact of those factors or to produce test scores which reflect the impact of such influences in any precise way. There is, therefore, a danger that unwarranted comparisons will be made

and unjustified conclusions drawn. International comparisons can and should be made, but the influence of important, relevant variables which might account for some or all of the differences that are found to exist must be considered. Two such variables, Opportunity To Learn (OTL) and Appropriateness, were utilized in the international study.

The pool of items developed for the study was divided among five test booklets: a Core Test and four rotated forms, called Forms A, B, C, and D. All students wrote the Core Test plus either one or two of the rotated forms. All British Columbia students were given the Core Test and one randomly assigned rotated form. As the study progressed, a few countries made changes in the assignment of items to forms, and some items were replaced. In the end, there were 157 items which were used in all 20 places which participated in the achievement testing at the Population A level. These are the items which will be discussed in the remainder of this chapter. Although the focus is on performance by British Columbia students, data from several other jurisdictions of particular interest are included to highlight the results. These are listed below along with the acronyms used to refer to them in tables.

Ontario	ONT
England and Wales	ENW
France	FRA
Japan	JPN
New Zealand	NZE
Sweden	SWE
United States of America	USA

Appropriateness of the Item Pool

Although every effort was made to ensure that the item pool would be equally fair for students in all countries, it was not possible to achieve this goal entirely. The results displayed in Table 6 show that, according to the Appropriateness Ratings assigned to each item by national committees in participating countries, there was a fairly wide degree of variation both within and between countries on the various subtests and on the pool of items as a whole.

The results show that, in all eight places, the vast majority of the 157 items were considered to be appropriate for students at this level. The most problematic area is geometry where there is a very wide spread: Only 44% of the geometry items were considered appropriate in British Columbia, while over 80% were considered appropriate in four of the other places. These Appropriateness ratings are indicators of the intended curriculum, and it will be clear from the OTL ratings provided by teachers that discrepancies exist between the intended and the implemented curricula.

Table 6
Appropriateness of the Item Pool
(Population A)

Subtest	No. of Items	% of Items Considered Appropriate							
		BC	ONT	ENW	FRA	JPN	NZE	SWE	USA
Arithmetic	46	96	100	98	83	93	94	87	100
Algebra	29	80	77	93	97	93	97	76	63
Geometry	40	44	87	87	54	87	87	49	59
Measurement	24	96	100	100	62	100	100	96	100
Descriptive Statistics	18	94	89	83	28	100	100	100	94
Total	157	80	91	93	69	94	94	78	82

Subtest Results

Analysis of achievement results in a comparative study such as this are of most value at the individual item level where analyses of errors and comparisons of performance levels can best contribute to an increase in our understanding of the learning process. In the following sub-sections each of the five major sub-tests will be examined in turn, with attention being focussed on particular items which have been selected to illustrate important points.

In general, results show that students found the items relatively difficult: the mean *p*-value for all 157 items and across the 20 participating countries was 47%. Students in a few countries did exceptionally well on a few items, but these were relatively rare. The histogram shown in Figure 3 summarizes the item-level results for the 3140 item scores obtained: 20 countries x 157 items. The graph shows comparatively small numbers of either very low or very high scores: the vast majority lie in the range between 30% and 70%. The standard deviation of this distribution was 20.

Population A Results

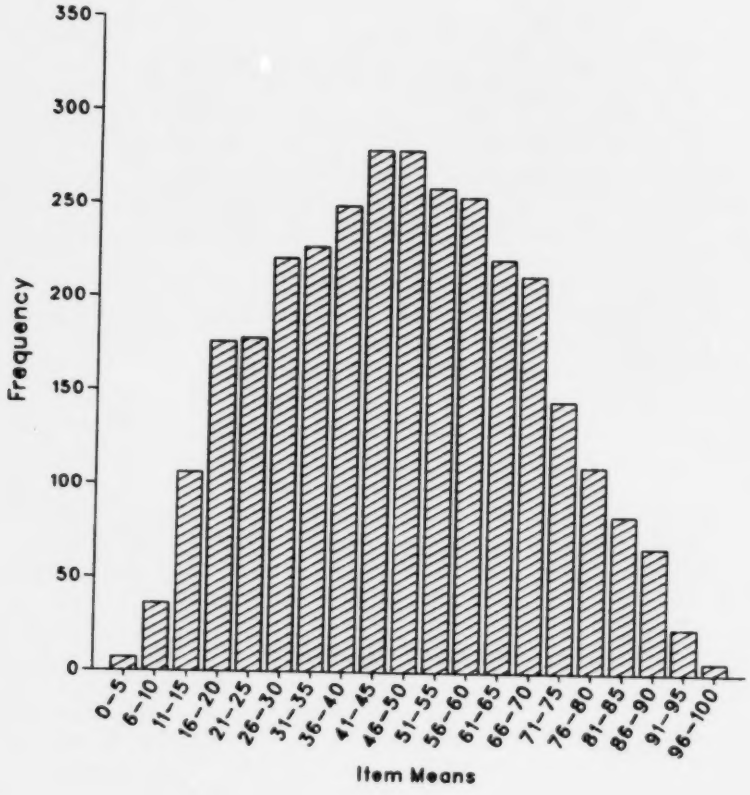


Figure 3. Distribution of item means: Population A.

Performance in Arithmetic

The graph in Figure 4 displays national mean scores on the 46-item Arithmetic subtest, along with the Opportunity-to-Learn scores for the same subtest. Results are displayed in descending order of achievement results, with the Japanese score being highest.

The OTL results show that, on the whole, the items on this subtest dealt with content which these teachers had either taught as new material or reviewed. Of the eight OTL scores shown, six exceed 75%. The Appropriateness ratings are similarly high. Over 80% of these items were rated as appropriate in every country. In British Columbia the OTL score for arithmetic was 87% and the Appropriateness score, 96. Similarly high ratings were given in Ontario, the United States, and Japan.

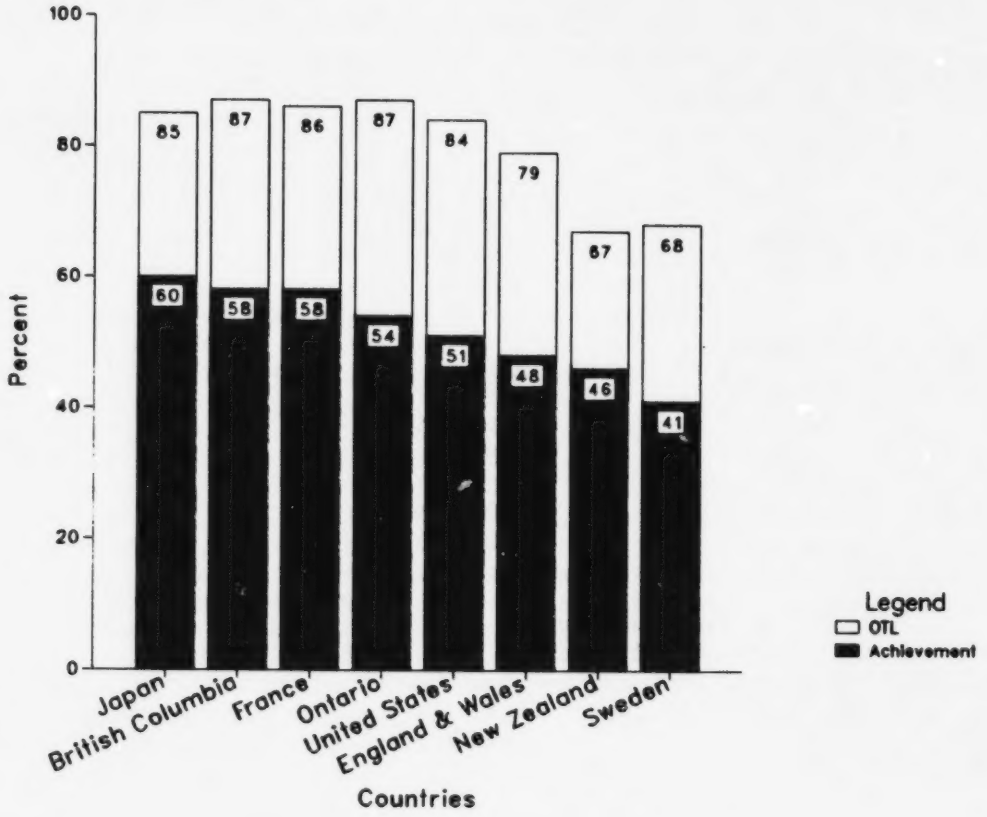


Figure 4. Performance in arithmetic

The items on this subtest dealt with a variety of arithmetic topics including whole numbers, common and decimal fractions, ratio and proportion, percent, number theory, exponential notation, and square roots. British Columbia students performed well on these items, obtaining a mean score of 58%. The highest score was 60%, in Japan, and the overall mean across countries was 50%.

Whole Numbers. This collection of items contained two of the ten items with the highest *p*-values in the item pool. Overall, 85% of students were able to find the product 162 x 45 (Item #073); and 78%, the difference 1054 - 865 (Item

#041). Students in France did particularly well on these two items, while students in Sweden did more poorly than the others. British Columbia results on these two items were 86% and 88% respectively; while those in Ontario were 86% and 87%, and in the United States 83% and 84%.

Students found some of the Whole Number items rather difficult. For example on Item #074, shown below, the median percent correct was 37%. The highest *p*-value on this item was 49%, obtained by students in England and Wales.

Item #074

1st row	1
2nd row	1 - 1
3rd row	1 - 1 + 1
4th row	1 - 1 + 1 - 1
5th row	1 - 1 + 1 - 1 + 1

What is the sum of the 50th row?

A	0 *
B	1
C	2
D	25
E	30

This item received very low OTL ratings in most places, including British Columbia at 26%. This is somewhat surprising since the arithmetic tasks involved are elementary. It may be that teachers felt that their students had never seen such an item and responded on that basis rather than in terms of the content involved.

Common and Decimal Fractions. On Item #003, which had a median OTL rating of 98%, students were required to find the sum of two common fractions:

$$\frac{2}{5} + \frac{3}{8}.$$

This appears to be a straightforward task with the two denominators being relatively prime, as are the numerator and denominator of the sum. However, the international mean percent correct for this item was only 58%, with a range from 89% correct in Japan to 25% in Sweden. Three of the countries being discussed in this report had *p*-values below 50%: England and Wales, 48%; New Zealand, 38%; and Sweden, 25%. In British Columbia, 68% of students gave the correct response and in Ontario, 62%. The United States' result was 57%.

The most common incorrect answer for this item was 5/13, a result obtained by adding the two numerators and then adding the two denominators. Such an error may result from students' confusing the addition algorithm for fractions with the multiplication algorithm. This incorrect response was selected by over 30% of students in England and Wales, New Zealand, and Sweden.

Performance on other items dealing with computation with fractions was no better. The mean percent correct on a subtraction item, Item #107,

$$1\frac{2}{5} - \frac{1}{2}$$

was 56%, while a division item, Item #139,

$$\frac{\frac{3}{5}}{\frac{2}{7}}$$

resulted in a mean percent correct of 38%. British Columbia scores on these items were 63% and 37% respectively. In Ontario they were 55% and 62%; in the United States, 56% and 61%.

Results on items involving computation with decimal fractions were also relatively poor. The mean percent correct on a multiplication item, Item #005,

$$0.40 \times 6.38$$

was 57%. In British Columbia, the *p*-value was 60%. In Ontario, it was 66%; in the United States, 60%. In this case, the most frequently chosen incorrect response was 25.52 which indicates that students either did not or could not verify the reasonableness of their answer.

On Item #108, which dealt with division of decimals, the mean percent correct was 41%. All five answer choices for this item contained the same three significant digits, and all that students had to do was choose the one which had the decimal point in the proper place. Scores in Ontario, the United States, and British Columbia were 61%, 60%, and 53% respectively.

Population A Results

Item #108

$$.004\overline{)24.56}$$

In the division above, the correct answer is:

A 0.614
B 6.14
C 61.4
D 614
E 6140 *

Ratio, Proportion, and Percent. Items dealing with ratio and proportion occurred in the Geometry subtest as well as in the Arithmetic subtest. These items dealt with topics such as expressing the relationship between two quantities as a ratio, setting up and solving proportional equations, indirect measurement, similar polygons, and using proportions to solve problems. Percent items dealt with the three cases of percent, and their application in the solution of practical problems. Students seemed to find the items in this set relatively difficult. In fact, the mean percent correct exceeded 50% on only 5 of the 13.

There was a considerable amount of fluctuation in the OTL results for these items. In general, items requiring the use or application of geometric concepts had significantly lower OTL scores than the other items, indicating that the study of similar figures is not part of the mathematics curriculum at this level in some countries. Several countries, notably New Zealand and Sweden, had low OTL scores on most of the items in this subtest and the scores obtained by students in those countries on those items were generally among the lower ones.

The most difficult ratio-and-proportion item involved finding the missing term in a proportion, and was in fact a very straightforward item.

Item #142

x	3	6	P
y	7	Q	35

The table above shows the values of x and y where x is proportional to y. What are the values of P and Q?

The highest percent correct achieved on this item was 50%, in Japan. The next highest score was a very low 26%. In Ontario and British Columbia, at 18% each, performance was below the chance level. In the United States, the p-value was 22%.

By far the most popular incorrect answer to this question was

$$Q = 10 \quad \text{and} \quad P = 31$$

This answer comes from noting that the difference between the original values of x and y, 3 and 7, is 4 and then finding Q by adding 4 to the second value of x, 6, and finding P by subtracting 4 from 35. It seems clear that many students have a less than adequate comprehension of the term *proportional*.

Other Topics. The topic of estimation is acquiring increased importance in the mathematics curricula of many countries. Mathematics educators believe that, as the use of calculators and computers grows, it will be increasingly important for students to be able to make estimates and judge the reasonableness of their results. As has already been seen in connection with an item involving division of decimals, many students do not appear to consider whether or not their responses make sense.

Only one item explicitly directed students to estimate, and they found it difficult. The item is shown below.

Item #045

The value of 0.2131 x 0.02958 is approximately:

A. 0.6
B. 0.06
C. 0.006 *
D. 0.0006
E. 0.00006

The item asked for the product of two decimal fractions which were expressed to several decimal places in order to discourage students from actually performing the calculation.

Population A Results

The most commonly selected incorrect response was E, which has the same number of decimal places as the multiplicand.

The mean percent correct on this item was 28%. Ontario students had the best performance of those in the eight jurisdictions being discussed here, 39%. The United States score was 30%; and the British Columbia one was 27%. In most places, except New Zealand, OTL levels for this item were very high. Only 46% of teachers in New Zealand said that their students had been taught this topic.

Performance in Algebra

The Algebra subtest consisted of 30 items dealing with topics such as integers, order of operations, simplification of algebraic expressions, evaluation of expressions and formulae, and solution of linear equations and inequalities. The graph in Figure 5 shows the level of achievement, along with the OTL results, for the eight jurisdictions on these 30 items. The international mean percent correct on this subtest was 43%.

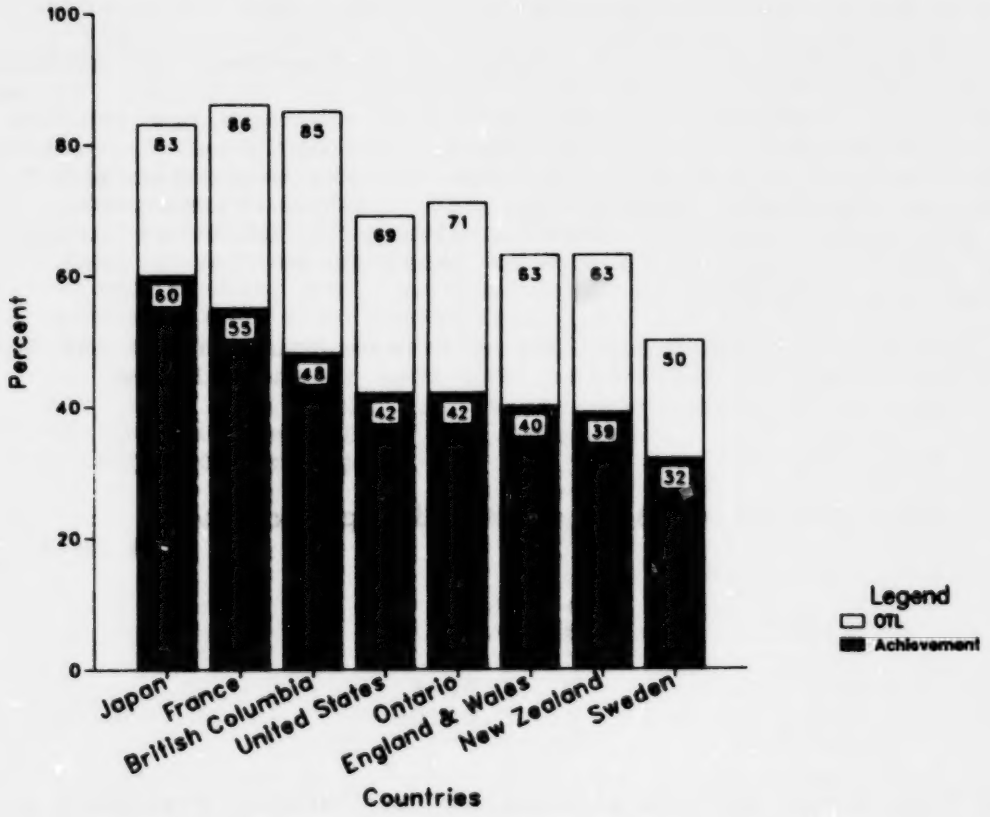


Figure 5. Performance in algebra.

On the whole the range of achievement scores here is very similar to that on the Arithmetic subtest. In fact, the countries whose students did best on this subtest were the same ones as on the former. The OTL and Appropriateness results, on the other hand, are generally lower; indicating that at least some of the topics covered by these items do not form part of the mathematics curriculum at the Population A level in some countries. The lowest Appropriateness ratings for this subtest came from Ontario and the United States.

In the analysis of the mathematics curriculum which was conducted as part of this study, it was found that England and Wales, France, Japan, and New Zealand emphasize the study of algebra at the Population A level. This is generally not the case in North America, except for a minority of students.

Although England and Wales, and New Zealand are included among the countries listed above as emphasizing algebra, their achievement results and OTL scores, as shown in Figure 5, are relatively low. On the other hand, British Columbia, which does not emphasize algebra at this level, has relatively high achievement results, and an OTL score of 85%. This points out one of the dangers involved in labelling a subtest such as this with a term which is open to a wide variety of definitions. It is likely that what is considered to be algebra in New Zealand, and England and Wales is different from the content covered by these items. On the other hand, these results indicate that British Columbia does include a fair number of algebraic topics in its curricula.

Integers and Order of Operations. Six of the 30 items on the Algebra subtest dealt with operations with integers and application of the standard rules for order of operations. On Item #049,

-5(6 - 4) is equal to

A	50	
B	26	
C	10	
D	-10	*
E	-26	

the mean percent correct was 61%, with a high of 78% in Japan and a low of 32% in Sweden. OTL scores for this item were over 80% in all countries except Sweden, where it was 45%. In British Columbia, the *p*-value was 76%. In Ontario, it was 63%, and in the United States, 62%.

On a word problem involving the use of integers, Item #013, students were required to find the difference between a temperature of -7 degrees and one of 31 degrees. OTL scores

on this item were high in all countries, but results were lower than those on the preceding item. The mean percent correct was 45%, with the British Columbia score being 58%; the Ontario one 51%; and the United States, 43%. The most common incorrect response was 24 degrees, which is the difference between the absolute values of the temperatures.

Simplification of Algebraic Expressions. Courses in Elementary Algebra typically include exercises such as Item #015,

$$\text{Simplify: } 5x + 3y + 2x - 4y$$

which require students to combine similar terms. Results on this item were as follows:

- mean percent correct: 48%
- highest percent correct: France, 78%
- lowest percent correct: Ontario, 17%
- British Columbia: 31%
- United States: 30%

The U.S.A. had the lowest OTL score, 49%, on Item #015, followed closely by Sweden at 50%. Almost all of the rest were fairly high with Ontario and British Columbia reporting 64% and 60%, respectively. The most commonly chosen incorrect response was,

$$7x + 7y$$

which is obtained by adding the coefficients of y rather than subtracting them.

Evaluation of Expressions and Formulae. On Item #017, students were required to find the value of W in the formula

$$P = LW$$

given that P = 12 and L = 3. This turned out to be one of the easiest items in the subtest, with a mean percent correct of 64% and a range from 89% in France to 46% in Sweden. North American results were 73% in British Columbia and Ontario, and 68% in the United States. OTL results were uniformly high except in Sweden where it was only 44%.

Students were much less successful at identifying a formula to fit a table of values as required in Item #055.

m	-1	1	2	4
n	-1	3	5	9

For the table shown, a formula that could relate m and n is:

A $n = m$
B $n = 3m$
C $n = -m^2 + 1$
D $n = m^2 + 1$
E $n = 2m + 1$ *

The highest *p*-value on this item was 49%, in Japan, while the lowest was 13%, in Sweden. The mean was 27%. All three North American scores were between 20% and 30%. OTL results were also much lower than on the previous item, with several countries reporting OTL scores below 50%. On the other hand the item was rated as appropriate in the majority of countries.

Solution of Equations and Inequations. Listed below are the wordings of three items dealing with the solution of equations and inequations, along with a summary of the results on each of them.

Item #	*Wording*	*Mean % Correct*
086	If $\frac{4x}{12} = 0$ then x is equal to	42
118	$\frac{x}{2} < 7$ is equivalent to	35
151	If $5x + 4 = 4x - 31$, then x is equal to	26

None of these items is particularly difficult and, at least the two involving equations, were considered appropriate in almost every country. Yet the performance levels are relatively weak on all three, and particularly on Item #151.

If such items are appropriate to the curriculum, if teachers say they have taught this material, and yet performance levels are so low, it may be the case that this material is beyond the capability of many Population A students.

Performance in Geometry

There were 39 geometry items in the pool of 157, 25% of the total. These dealt with a wide variety of topics such as properties of plane figures, similarity and congruence, the Pythagorean theorem, elements of proof, formal and informal transformations, coordinate geometry, spatial visualization, and elementary properties of figures in 3-space. The very variety of topics speaks to the lack of consensus that exists both between and within countries concerning appropriate geometric content, as well as to the importance of geometry in the mathematics curriculum.

It was extremely difficult to assemble a group of geometry items which all participants agreed were appropriate to their curricula. The geometry taught ranges from elementary properties of common plane figures in some countries through an extensive range of geometry topics in others. It is based on a traditional approach in most countries, but on an approach through transformations in others. Teaching methods range from informal and intuitive to rigorous and axiomatic. Every effort was made to accommodate this diversity, by including as many items as possible which were amenable to solution by either traditional or transformation methods. The final set of items contained 12 items dealing with transformational geometry. Eight of these were classified as informal transformations, and the remaining four as formal.

The formal transformation items proved to be too difficult for Population A students everywhere. No country had a *p*-value greater than 50% on any of these items, and the mean percents correct for the four were 23%, 22%, 18%, and 14%. In France, which supplied the items, results were as low as elsewhere. OTL scores were also universally low.

The confusion and lack of consensus about the teaching of geometry is reflected in the graph of results from the Geometry subtest shown in Figure 6.

First of all, the OTL scores are remarkably low. In all of these countries, it is less than 60%. In other words, teachers in these countries felt that less than 60% of the items on this subtest dealt with content which was part of their curriculum at the Population A level or lower. This situation is complicated by the fact that the items making up the 60% or more which were taught varies from one country to the other, making it difficult to find many items which are comparably appropriate across countries.

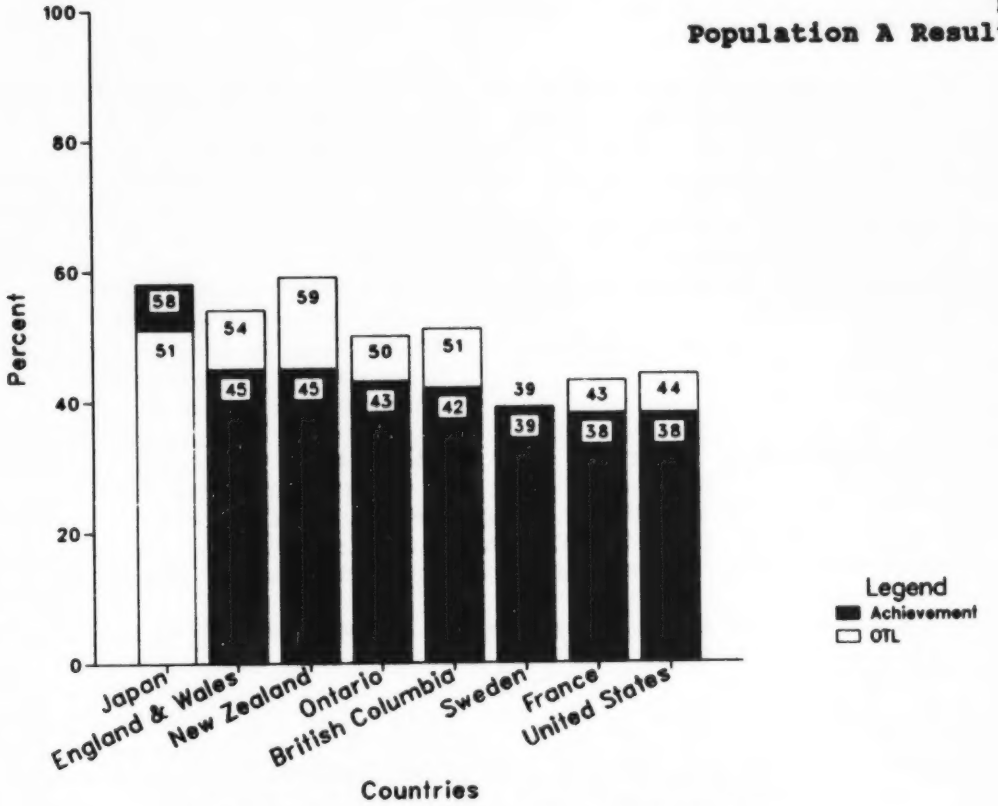
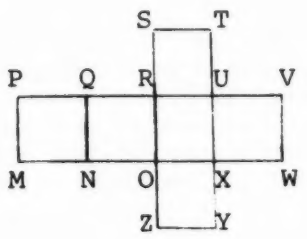


Figure 6: Performance in geometry.

In Japan, the OTL score was lower than their achievement score on the subtest; in Sweden they were equal. In most other countries the two were much closer to each other than was the case on either the Arithmetic or the Algebra subtests. This may be partially accounted for by the inclusion of several items, particularly on the topic of spatial visualization, which students might have been able to solve on the basis of prior experience and knowledge of their environment. An example of this occurred in Item #031.

Item #031



Population A Results

> The diagram shows a cardboard cube which has been cut along some edges and folded out flat. If it is folded again to make the cube, which two corners will touch corner P?
>
> A corners Q and S
> B corners T and Y
> C corners W and Y
> D corners T and V *
> E corners U and V

The mean percent correct for this item was 62%, but more interesting is the fact that in most of these countries, including the three North American jurisdictions, students' performance exceeded the OTL score. In British Columbia, for example, the *p*-value was 74% and the OTL score, 28%.

On the whole, and as the graph in Figure 6 shows, performance levels on this subtest were not high in these countries. They ranged from a low of 38% in the United States, where the OTL score was only 38%, to a high of 58% in Japan. On only 6 of the 39 geometry items was there a definite consensus among teachers from almost all countries that these items dealt with geometric content which was appropriate to their students. These items--#022, #028, #057, #089, #121, and #122--dealt with definitions and elementary properties of plane figures. For example, on Item #121 students were asked to decide which of five angles shown was acute. The median percent correct for that item was 61%, with a high of 76% in Ontario. In British Columbia and the United States, the *p*-values were 57% and 52%, respectively.

Taken together, the results on this subtest underscore the urgent need for action to be taken to clarify and improve the status of geometry in the mathematics curriculum. In 1959 at the Royaumont Conference hosted by the OECD, the French mathematician Jean Dieudonné (1961) told us that "Euclid must go!" More than two decades later the situation is one of confusion and lack of direction. Much of what used to be taught is gone, but attempted replacement of that content with topics from transformational geometry and linear algebra appears to have been less than successful internationally. Geometry is an important branch of mathematics, and a person's mathematical education would be incomplete without the inclusion of a coherent, serious treatment of geometry. In many countries it appears that, unless some action is taken soon, geometry may well disappear from the mathematics curriculum except for vocabulary and a few elementary topics treated in an informal, intuitive way.

Performance in Measurement

The 24 items on the Measurement subtest dealt with students' knowledge of the basic units of measurement in the metric system, finding areas and volumes, and estimation. The graph in Figure 7 shows relatively high OTL scores in most countries, and this indicates that this material is part of the mathematics curriculum of most countries. The mean score on the Measurement subtest was 51%, with a high of 69% in Japan, and a low of 45% in the United States.

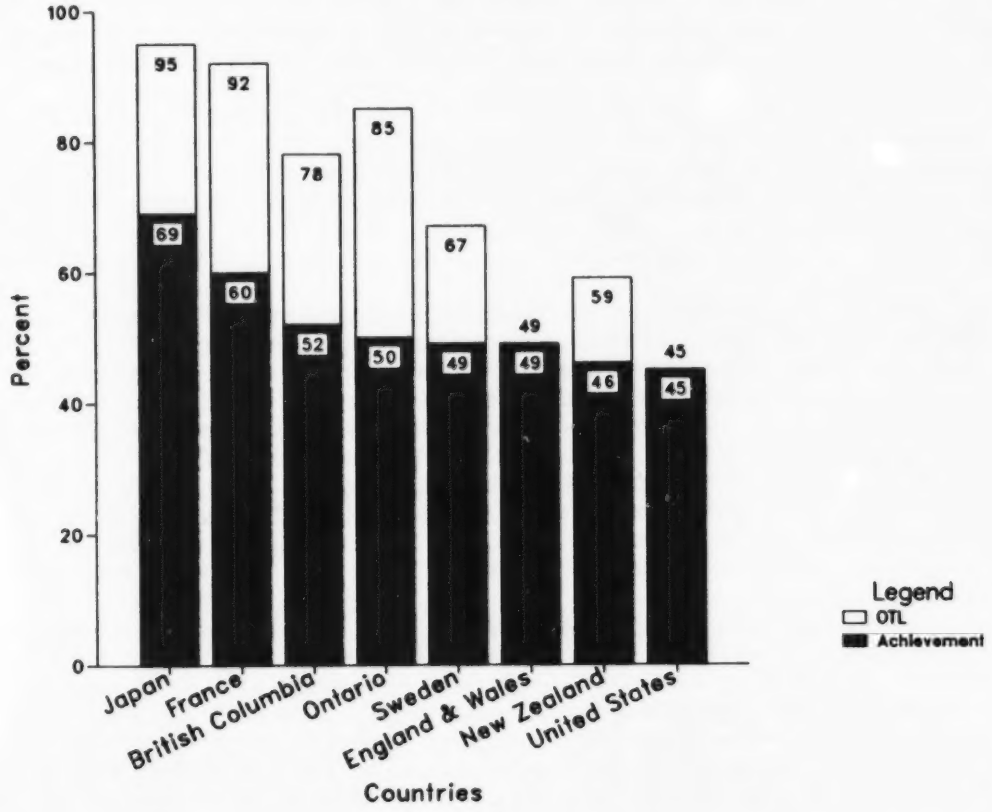


Figure 7. Performance in measurement.

Metric Units. Most of the 20 countries which participated in the international study have made use of the metric system of measurement for many years. On the other hand, most of the English-speaking countries which participated have changed from the British or Imperial system of measurement to the metric system relatively recently. The United States was the only country participating in the study which had not officially adopted the metric system.

These differences show up very clearly in the item-results. In a metric country such as France, Item #036, which asked students to select the most likely weight[1] of a normal man, would probably not be considered a mathematics item at all: 98% of French students gave the correct response. In the United Sates and several other countries, on the other hand, the metric system is taught as part of the mathematics curriculum at this and earlier grade levels. Only 35% of United States students obtained the correct answer to this item, the lowest correct result among all participants.

In general the results on these items reflect satisfactory levels of performance. Countries which have been metric for a long time tend to do better on these items than those who have adopted that system more recently. For example, on the item mentioned above, over 20% of the students in each of the English-speaking country chose 185, the correct answer in avoirdupois units, rather than 85 as their response. For students in Canada, New Zealand, England and Wales, and the United States, 185 pounds is more familiar than 85 kg. Students in these countries do not, as yet, "think metric."

Area and Volume. In general, students did well on items which called for straightforward application of a rule or formula. On the other hand, items which required the use of higher order thinking skills resulted in lower levels of performance. For example, on Item #039 students were given the dimensions of a rectangular prism and asked to find its volume. The mean percent correct was 69% and only three countries had *p*-values below 60%: New Zealand, 50%; United States, 49%; and Sweden, 48%. In British Columbia, the *p*-value was 69%; in Ontario, 64%.

When asked, in Item #070, to find the area of a parallelogram given the base, the altitude, and the slant height, however, the mean percent correct fell to 34%. This time only the Japanese *p*-value exceeded 60%. In fact, it was the only one that exceeded 50%. The most common incorrect response to this item was that obtained by finding the product of the base and the slant height. All three groups of North American students had identical scores: 35% correct.

Estimation. Students found one of the estimation items in the Measurement subtest easy. On Item #069 they were shown the length of a unit and asked to estimate the length of a given segment. The mean percent correct on this item was 84% with a high of 96% in France and British Columbia, and a low of 74% in England and Wales. The Ontario result was 93%, and the United States, 86%. However, on Item #102 which required them to estimate the number of degrees in an acute angle, the mean fell to 50%. Similarly on Item #072, in which students

---

[1] The term "weight" was used rather than "mass".

were given the dimensions of a rectangular box expressed to the nearest tenth of a centimetre and asked to estimate its volume, the mean percent correct was 43%. In British Columbia, scores on these two items were 44% and 41%, respectively.

Performance in Descriptive Statistics

International and nationally-based groups in a number of countries interested in the advancement of mathematics education have urged teachers to include fundamental ideas from the areas of probability and statistics in the mathematics curriculum, beginning with the earliest school years. The Descriptive Statistics subtest consisted of 18 items which were intended to measure students' knowledge and ability in areas such as reading information from charts, tables, and graphs; calculating averages; and, in the case of one item, finding the probability of an event. The graph in Figure 8 summarizes the results of this subtest.

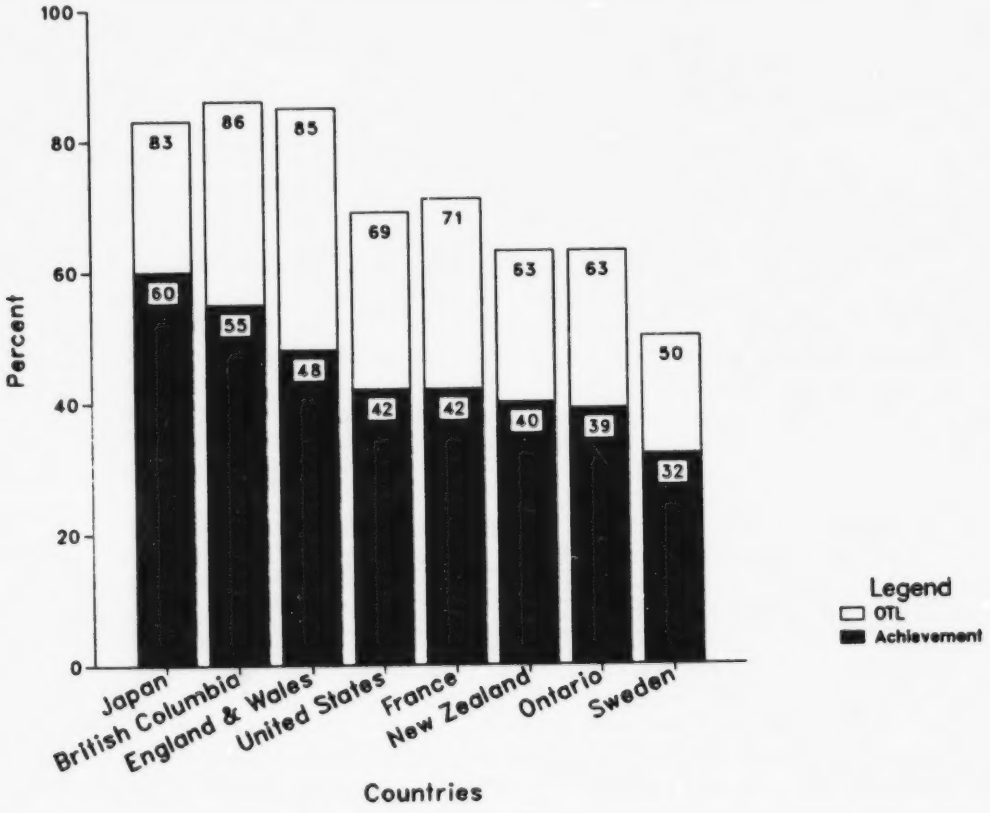


Figure 8. Performance in descriptive statistics.

Population A Results

In a few countries other than those shown here, students outperformed their teachers' OTL predictions on this subtest. That is almost certainly attributable to the fact that students gain experience with charts, tables, and graphs in contexts other than their mathematics classes. Thus, on Item #033 which required students to interpret information presented in a circle graph, the mean percent correct was 82%, and *p*-values in each of these 8 places all exceeded 80%. In British Columbia, where the *p*-value was 89%, the OTL score was only 59%. Students know how to read information from circle graphs, but their mathematics teachers apparently do not teach them to do so.

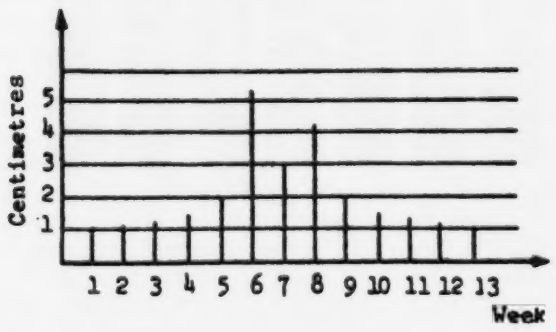
Uniformly high performance was recorded on Item #067:

> Joe had three test scores of 78, 76, and 74, while Mary had scores of 72, 82, and 74. How did Joe's average compare with Mary's?

The mean percent correct in this item was 77%, but the range was only 18 points. France had the highest score, 89%. OTL scores for this item were high almost everywhere. British Columbia students had a mean percent correct of 78%. In Ontario and the United States, the scores were 76% and 73%, respectively.

One of the most difficult items in this subtest required students to estimate the arithmetic mean of 13 data points shown on a graph. The item, Item #099, is shown below:

Item #099



In the graph, rainfall in centimetres is plotted for 13 weeks. The average weekly rainfall during the period is approximately:

A 1 cm
B 2 cm *
C 3 cm
D 4 cm
E 5 cm

The mean percent correct was 42% with a high of 67% in Japan and a low of 41% in Sweden. Students in British Columbia obtained a score of 50%.

General Comments

In order to compare performance levels across subtests *z*-scores were computed, and these are presented in Table 7.

The entries in the table highlight the curricular differences which exist among these jurisdictions. For example, students in British Columbia and Ontario performed best on the Arithmetic subtest, while those from England and Wales, Japan, and Sweden had their poorest performances on that same set of items. Corresponding differences occurred on all five subtests. In other words, there is no one subtest where students from all countries performed in the same overall fashion.

It is interesting to note, in connection with the Geometry subtest, that students in France were among those who had their poorest performance on that subtest. If the study of geometry constitutes a significant portion of the mathematics curriculum, then these results are an indication of the lack-of-fit between the French geometry curriculum and geometry as defined by the set of items used in this study. This finding underscores the importance of interpreting these achievement results cautiously. They are a valid basis for drawing comparisons only insofar as the items which define the subtests are equally appropriate to the curricula of the countries being compared.

Table 7
Subtest Comparisons: *z*-Scores

	Arithmetic	Algebra	Geometry	Measurement	Descriptive Statistics
British Columbia	1.0*	0.6	0.1**	0.1**	0.7
Ontario	0.6*	-0.1**	0.2	0.0	0.2
England & Wales	-0.3**	-0.3**	0.5	-0.2	0.6*
France	1.0	1.4*	-0.4**	1.0	0.3
Japan	1.2**	1.9	2.1*	2.0	1.8
New Zealand	-0.6**	-0.4	0.5*	-0.6**	0.3
Sweden	-1.2**	-1.2**	-0.3	-0.2	0.2*
United States	0.1	-0.1	-0.4	-1.1**	0.4*
Int'l. Mean	50.	43.	41.	51.	55.
Std. Dev.	7.6	8.8	8.1	9.1	9.3

* - *Best subtest performance for that country*
** - *Poorest subtest performance for that country*

Easiest and Most Difficult Items

International mean scores for the 157 items in the pool ranged from a high of 84% on Item #069, which asked students to estimate the length of a given segment, to a low of 13% on Item #173, which involved identifying the difference of two vectors. Sixteen items, 10% of the total, had *p*-values of 70% or more. At the other extreme, thirteen items resulted in *p*-values of 25% or less, including four whose *p*-values were less than 20%, the chance level.

Of the 16 easiest items, five were computation items; three involved the use of metric units of measurement; and two, the ability to read information from graphs. On the whole, the items which students found easiest were those that required them to recognize a definition or rule, or to apply a standard computational technique.

Of the 13 most difficult items, six were from the Geometry subtest, and four of these dealt with transformational geometry in a relatively formal way. Among the remaining most difficult items, two dealt with finding the surface area or volume of a rectangular prism; one with finding missing terms in a proportion; and one with recognizing the application of the associative and distributive properties within the set of whole numbers.

In general, and with the exception of the transformational geometry items which had very low OTL scores, the difficult items required students to read more text, or to solve a problem which was somewhat different from a routine application. An interesting contrast is seen in the results of two items dealing with graphs. On Item #033 students were asked to read a piece of information from a circle graph, while, in Item #129, they were asked to determine the size of the central angle for a particular segment of a circle graph. On the former, the mean *p*-value was 79%; on the latter, 22%.

These achievement results raise a number of questions and concerns. For example, on items testing computation with decimals, fractions, and percent, performance internationally can be described as satisfactory at best, and in fact was somewhat disappointing. Students in some countries consistently outperformed those in other countries, but those results can only be interpreted in a meaningful way at the national level. It remains for each of the National Centres to analyze these data in the light of relevant socio-economic factors as well as the influence of the OTL and Appropriateness variables reported here and, on that basis, to draw their own conclusions about the levels of performance attained by their students.

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Population A Results

CHAPTER 3

Population B Results

The makeup of Population B varied dramatically among the countries participating in the study, and it is, therefore, necessary to be extremely cautious in making comparisons and drawing conclusions on the basis of these results. At one extreme we have the case of British Columbia where Population B represents 30% of the age cohort; at the other extreme, in England and Wales Population B represents just 6% of the age cohort. Not only must differences in the ability of the students making up these groups be taken into account, but also the fact that the mathematics courses offered to a highly select group will be very different from the Algebra 12 course offered to our Grade 12 students. For example, in most countries participating in this study, Population B students take a full year of calculus in the last year of secondary school.

The structure of the item pool for Population B reflects the emphasis given to the study of calculus in many countries at this level. Of the 136 items used, 46 were included in the Elementary Functions and Calculus subtest and, of these, 27 dealt specifically with calculus. Needless to say, students in British Columbia did not do very well on these items since calculus is not part of the Algebra 12 curriculum. In Ontario, on the other hand, the international Population B level sample was drawn from among Grade 13 students who were taking at least two mathematics courses, including calculus.

Fortunately, there are several sources of valid comparison for the British Columbia Population B results. The United States sample, which consisted of Grade 12 mathematics students as did British Columbia's, may be broken into two parts: one consisting of students following a pre-calculus course similar to our Algebra 12, and another, much smaller, taking Advanced Placement Calculus. The former, which will be referred to as the USA-Pre Calculus group in the remainder of this chapter, constitutes about 95% of the United States Population B; the latter, USA-Calculus, about 5%.

Comparing British Columbia Algebra 12 students' performance to USA-Pre Calculus scores is somewhat unfair, since the 5% of students thereby excluded from the American sample undoubtedly represents some of their best students. For that reason, the results of the entire United States Population B sample, USA-Grade 12, will also be reported. None of these three American groups is entirely comparable to British Columbia's Population B, since together they represent only 12% of the age cohort compared to our 30%. However, the content of the intended curriculum of the US-Pre Calculus

group is virtually identical to that taken by our Algebra 12 students, and some cautious comparisons may therefore be made.

A second and better source of comparison is the Ontario Grade 12 sample, which is referred to as Ontario-Grade 12 in the rest of this chapter. This group represents 30% of the Ontario Grade 12 age cohort, the same as our Algebra 12 group, and their curriculum appears to be very similar to that of Algebra 12. Students in the Ontario-Grade 12 sample take a course called *Foundations of Mathematics* which includes the following topics: geometry, relations and functions, quadratic functions and equations, analytic geometry, trigonometric functions, exponential and logarithmic functions, sequences and series, mathematical induction, and probability and statistics. Details on the specific content included in each of these strands is not available.

In addition to the groups of North American students, data from four other countries--England and Wales, Japan, New Zealand, and Sweden--are also discussed in this chapter. All three of these also participated at the Population A level. Unfortunately, France, which did participate at the lower level withdrew from the Population B study.

Opportunity-to-Learn Ratings

As might be expected, given the vast differences in the make-up of Population B in different countries and in the curriculum offered to those students, the Opportunity-to-Learn results fluctuated over a broad range both between subtests within countries and across countries. The OTL results for six of the Population B subtests are summarized in Table 8. Appropriateness ratings have not been included, since they were unavailable for several of the North American sub-populations.

The OTL results demonstrate clearly that much of the content of the Population B item pool was not part of the implemented curriculum in British Columbia. This was particularly true for the Elementary Functions and Calculus subtest and the Probability and Statistics subtest. In fact, the only subtests which really appeared to fit the British Columbia Algebra 12 curriculum were Algebra and Number Systems.

The fit of the item pool to the curriculum was much better in England, Japan, New Zealand, and, to a lesser degree, Sweden. These countries retain a much smaller proportion of their age cohort in pre-university mathematics and teach them more advanced topics than is the case in courses at this level in North America, even in Calculus courses. The Ontario-Grade 13 results seem too low, given the fact that these students were taking two Grade 13 mathematics

courses, including calculus. In fact, the way the OTL question was worded on the questionnaire made it difficult to interpret the results unambiguously. The Ontario OTL data at this level should, therefore, be interpreted cautiously.

Table 8
Opportunity-to-Learn Ratings
(Population B)

	Subtest					
	Sets	No. Syst.	Alg.	Geom.	Func./ Calc.	Prob./ Stats.
BC-Grade 12	66	75	83	51	32	28
Ontario-Grade 12	59	55	78	40	22	18
USA-Pre Calculus	81	81	86	61	45	43
USA-Grade 12	83	83	88	62	54	45
USA-Calculus	90	89	94	67	91	56
Ontario-Grade 13	54	56	78	45	39	30
England & Wales	53	75	87	69	87	70
Japan	95	80	100	90	92	83
New Zealand	85	90	93	74	93	86
Sweden	62	88	90	66	63	80

Subtest Results

The 136 achievement items were partitioned among 8 test booklets, 17 to a form, and students were allowed 50 minutes to complete a booklet. In most places each student wrote two of these rotated forms; but in some, including British Columbia, each student wrote only one.

All of the items were presented in multiple-choice format with five response choices, and the mean percent correct across all items and countries was 47%, the same as at the Population A level. As might be inferred from the fact that there were only 17 items on each form, many of the items were difficult, non-routine, and required students to use higher order thinking skills. Shown below, by way of example are two of the most difficult items in the pool: Items #085 and 127. The international *p*-values for these items were 22% and 16%, respectively.

Item #085

In any triangle ABC, if *a*, *b*, and *c* are the lengths of the sides opposite angles A, B and C respectively, the following relationship is always true:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

If b, c and angle B are given, with b<c and angle B acute, which of the following MUST be true to give unique solutions for the measures of angle A and angle C?

A $b = c \sin B$ *

B $\frac{\sin A}{\sin B} < 1$

C $b \leq a$

D $\sin B > \frac{b}{c}$

E $b \leq a$ and A is an acute angle

Item #127

$5^{2n} + 5^n$, where n is a natural number, is divisible by 13

A when $n = 2$ only
B when n is a non-negative even number only
C when n is of the form $8p + 2$ only, where p is a natural number
D when n is of the form $4p + 2$ only, where p is a natural number *
E for no value of n

Sets, Relations, and Functions

There were 7 items on the Sets, Relations, and Functions subtest dealing with topics such as reading information from a Venn diagram, set operations, and domain and range of a function. The international mean on this subtest was 62%, the highest of the six subtest scores to be discussed here.

The graph shown in Figure 9 shows the mean subtest score on the Sets, Relations, and Functions strand for the ten groups of students being discussed. The width of each bar is proportional to the size of the group as a fraction of the corresponding age cohort. Thus, the BC-Grade 12 and Ontario-Grade 12 bars are of equal width since they each represent 30% of the age cohort. The USA-Grade 12 bar is 0.4 times as wide as the BC-Grade 12 bar, since the USA-Grade 12 group represents 12% of its age cohort.

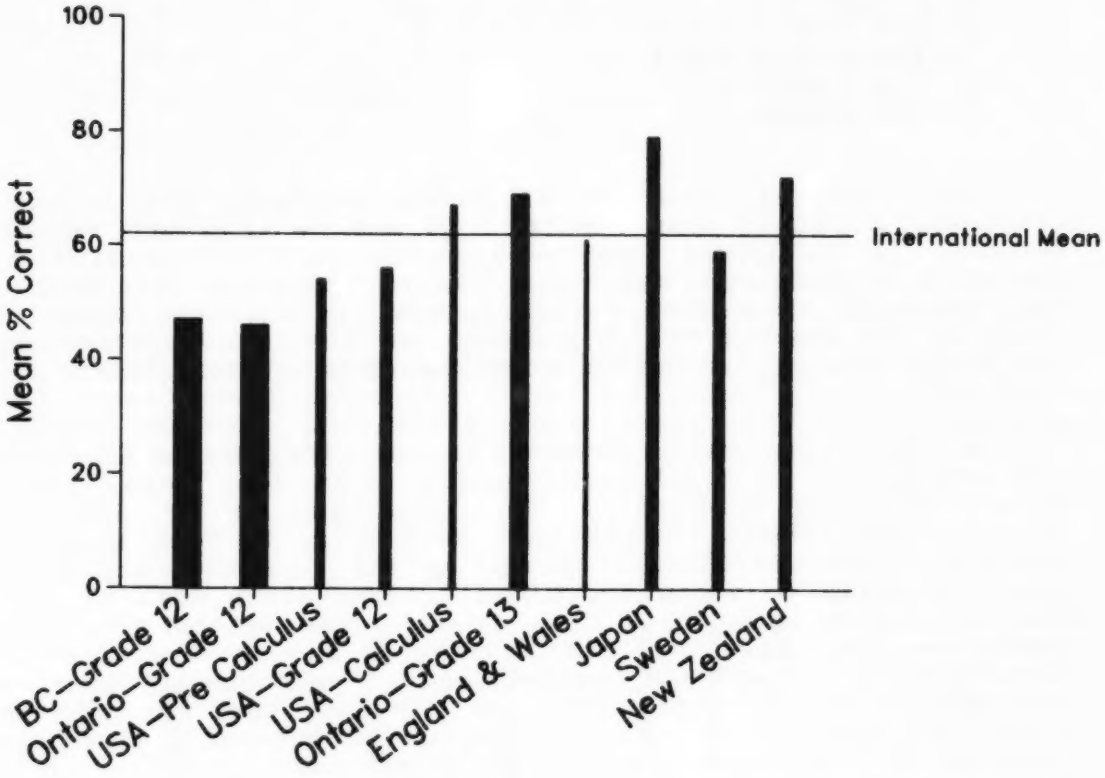


Figure 9. Performance in sets, relations, and functions.

The means for the USA-Calculus and Ontario-Grade 13 groups were 67% and 69%, respectively, several percentage points above the international mean. The scores for the other four groups were considerably below that level. BC-Grade 12 students achieved a mean score of 47%, almost the same as the 46% attained by Ontario-Grade 12 group. Students in the USA-Pre Calculus and the combined USA-Grade 12 groups did much better on these items than the Canadian students did.

Item #016, shown below, was included in this subtest, and it resulted in rather disappointing levels of performance. Students were given the definition of a function and asked to select the correct description of its graph.

Item #016

Let a function f be defined by $f(x,y) = x$.
Let $G = \{x,y \mid f(x,y) = 2\}$.
The graph of G is

Population B Results

A the line $x = 2$ *
B the line $x + y = 2$
C the line $x = y$
D the x-axis
E the y-axis

Only 58% of BC-Grade 12 students chose the correct response, a rather poor result. It may be that set-builder notation is no longer used very extensively by teachers or included in textbooks and, as a result, students were unable to understand what they were required to do. The Ontario-Grade 12 and USA-Pre Calculus scores on this item were 56% and 51% respectively, indicating that they had similar difficulty with the item.

Another item which students found difficult dealt with the concept of an additive function. On Item #076, shown below, students were given the definition of an additive function and asked to apply that definition to four specific examples. The USA-Calculus group, at 45%, was the only North American group to score above the international mean of 43%, and most were well below that level. The British Columbia score was 29%, while the Ontario-Grade 12 and USA-Pre Calculus results were 23% and 37%, respectively. Undoubtedly, part of the difficulty with this item was the fact that the correct answer was Response E: *None of the above*. However, given the familiarity of the functions specified in the four distractors, this is a disappointing result. It seems clear that students, even those who have taken a year of calculus, still have difficulty dealing with functional notation, and abstract concepts concerning functions.

Items #076

A real valued function f defined on a set of real numbers is said to be additive if, for all x and y in the domain of f,

$$f(x + y) = f(x) + f(y)$$

If the function f defined on the set of positive real numbers is

A $f(x) = x^2$, then f is additive.

B $f(x) = \sin x$, then f is additive.

C $f(x) = \log_{10} x$, then f is additive.

D $f(x) = 2^x$, then f is additive.

E None of the above is additive. *

Number Systems

The 17 items on this subtest dealt with properties of the rational, real, and complex number systems. Included were items concerning order of operations, the commutative property of multiplication, exponential notation, repeating decimals, and least common multiple. The international mean on this subtest was 50%, and, as the graph in Figure 10 shows, all six North American groups scored below that level.

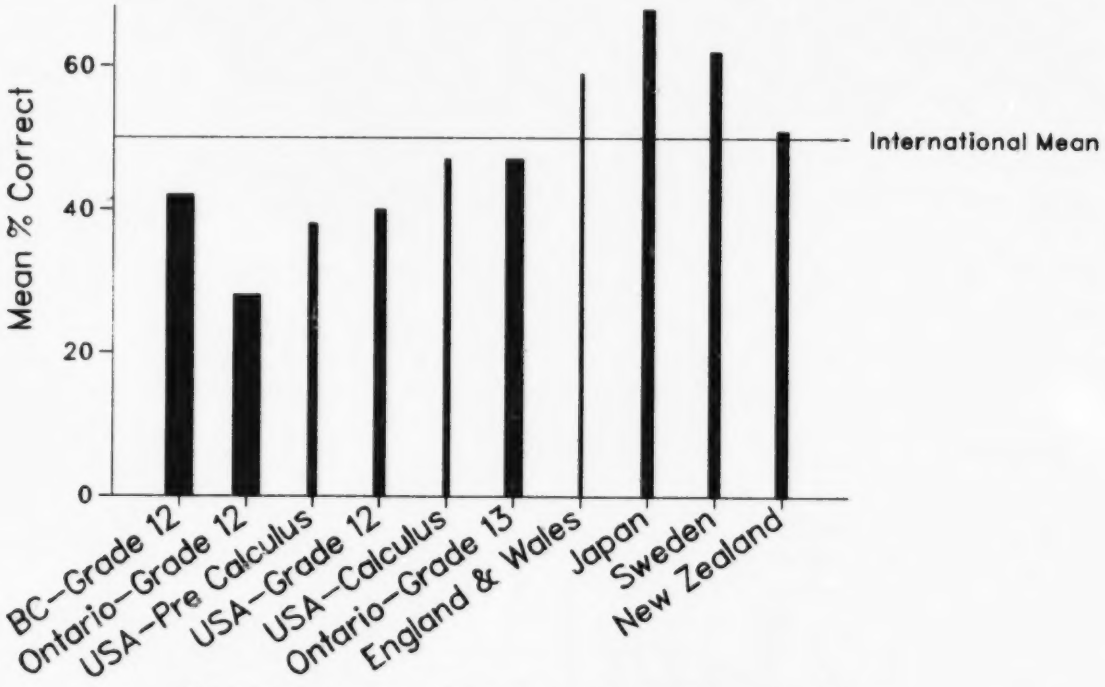


Figure 10. Performance in number systems.

On Item #077 students were asked to solve a simple exponential equation. All the North American groups performed well on this item which proved to be one of the easiest of the 136 used in the study: the international mean percent correct was 81. BC-Grade 12 students obtained a score of 72%, while their counterparts in Ontario and the United States had 66% and 70%, respectively.

Population B Results

Item #077

If $3^{x+y} = 81$ and $25^{x/2} = 5$, then y is

A 0
B 2
C 3 *
D 7/2
E 15/4

Lower levels of performance occurred on Item #092, a straightforward question on complex numbers. The international mean for this item was 61%, so British Columbia students, at 74%, did particularly well. The introduction of complex numbers does not occur as early in some other places as it does here, and that fact undoubtedly explains the relatively low international mean. The Ontario-Grade 12 result on this item was 28% correct; the USA-Pre Calculus, 55%.

Item #092

The complex number $(1 + i)^2$ is equal to

A 0
B 2
C $2i$ *
D $1 + i$
E $2 + 2i$

Algebra

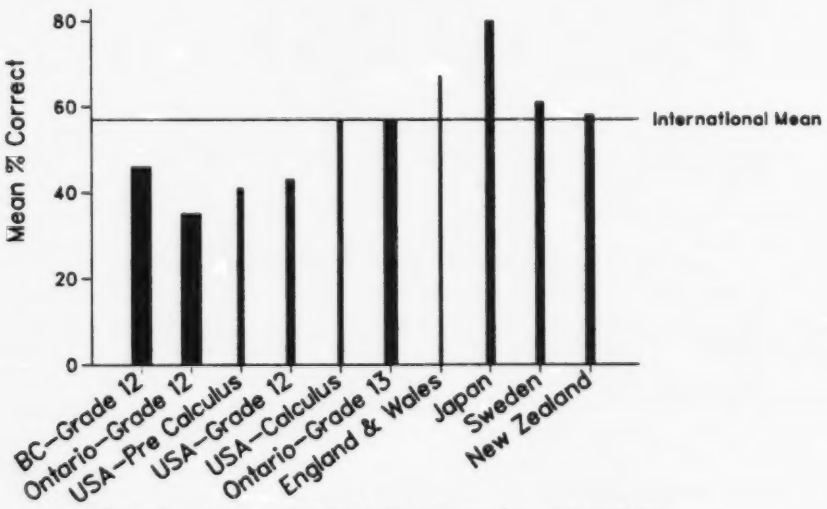


Figure 11. Performance in algebra.

There were 26 items on the Algebra subtest dealing with many of the topics traditionally found in an algebra course at this level. These included solving equations, both linear and quadratic, simplifying rational and radical expressions, constructing an expression or equation as a mathematical model of a situation, and so on. The international mean on this subtest was 57% and both the US-Calculus and Ontario-Grade 13 groups attained that level. As is shown in Figure 11, the other North American groups obtained significantly lower scores. BC-Grade 12 students out-performed their counterparts from the USA-Pre Calculus and Ontario-Grade 12 groups.

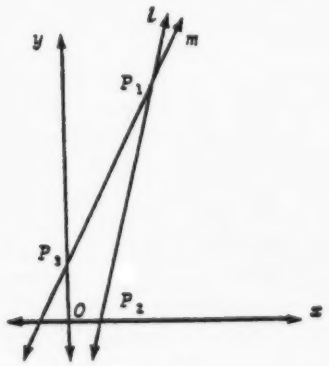
The item with the highest international mean score in the entire item pool was Item #082, an algebra item. The international mean percent correct on that item was 90%, and BC-Grade 12 students had a score of 88%. Admittedly this was an easy item, but it dealt with a topic of fundamental importance in secondary school mathematics, and it is heartening to see such high levels of performance on such a topic. Scores for Ontario-Grade 12 and USA-Pre Calculus were 77% and 78%, respectively.

Item #082

For the equations $x^2 - 5x + 6 = 0$

A there is no solution
B there is exactly *one* solution
C there are exactly *two* solutions *
D there are exactly *three* solutions
E there are *more than three* solutions

Item #037



The equation of line l is $y = 4x - 5$.

The equation of line m is $y = 2x + 2$.

Population B Results

What is the solution of the simultaneous equations

$$y = 4x - 5$$

$$y = 2x + 2$$

A The coordinates of P_1 *
B The coordinates of P_2
C The coordinates of P_3
D The x-value at P_2 and the y-value at P_3
E The y-value at P_2 and the x-value at P_3

Another item which would appear to be as easy and as important as the previous one was Item #037; but, at 80%, the international mean score on this item was considerably lower than that for Item #082. BC-Grade 12 students had a score of 75% correct, and this compares favourably with the international result. The performances of the Ontario-Grade 12 and USA-Pre Calculus groups, 48% and 57% respectively, were much weaker. The content of this item is treated in first-year algebra courses, usually no later than Grade 9, and it is surprising to see that students were not able to obtain a better result on such an elementary topic. It may be that not enough attention is given to graphical interpretations of algebraic situations and, if that is the case, steps should be taken to ensure that graphs are used more extensively.

Geometry

If it is difficult to identify a common international core of geometric content for the Population A curriculum, it is even more so at the Population B level. On the one hand, the field is much broader at this level, and the pool of geometry items for this study included both analytic geometry and trigonometry. On the other hand, in certain countries the approach taken to the study of geometry is fundamentally different from that to which we are accustomed in North America. In countries such as Belgium and France, the geometry curriculum is built around the study of geometric transformations, and that content would be completely unfamiliar to almost all North American students and to many of their teachers. Consider for example, Item #130.

The international mean on this item was 23%, and the British Columbia result was 15%. North American students in general did very poorly on all such items. It is also true, however, that performance on these items was poor even in Belgium where, according to the Appropriateness and OTL data, items such as this dealt with content from the intended curriculum.

Item #130

Let f be the mapping from the complex plane C to C which maps z into z' such that $z' = (1-i)z + 4$.

Which of the following describes f?

A f is a translation.

B f is a clockwise rotation about (0,-4) through an angle $Pi/4$.

C f is an enlargement of scale factor 2 and centre (0,-4).

D f is an enlargement of scale factor 2 and centre (0,4) followed by a clockwise rotation through an angle of $pi/4$.

E f is an enlargement of scale factor 2 and centre (0,-4) followed by a clockwise rotation through an angle of $pi/4$. *

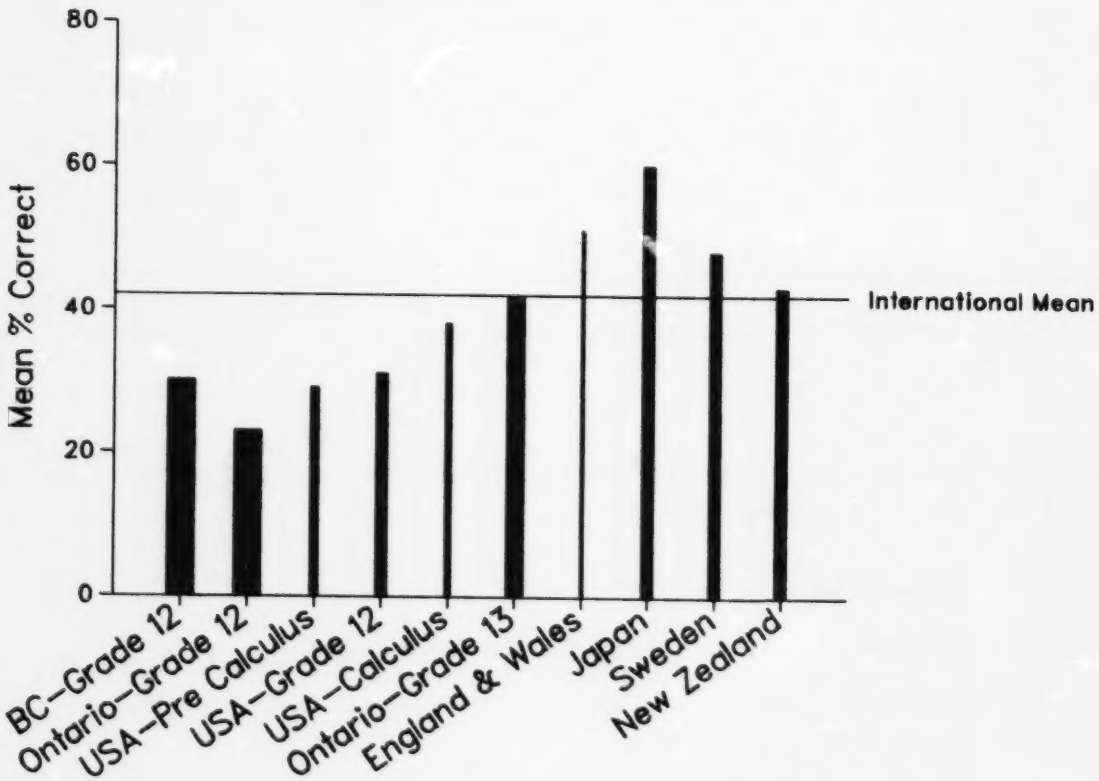


Figure 12. Performance in geometry.

Population B Results

There were 26 items on the Geometry subtest, and the international mean percent correct was 42%, the lowest of the six subtest means. The Ontario-Grade 13 group was the only one of the North American groups whose score reached the international mean; all the rest had considerably lower scores. The results are summarized in Figure 12.

Students did not do very well on any of these items, including a number which were well suited to the North American curriculum. On Item #009, for example, the international mean was 71%, but British Columbia students had a score of only 56%. The scores of the Ontario-Grade 12 and USA-Pre Calculus groups, at 39% and 49% respectively, were also disappointingly low on this item, which is an elementary application of the point-slope form for the equation of a line. British Columbia students should have been introduced to this notion in their Grade 11 course, if not earlier.

Item #009

In a Cartesian coordinate system, what is the equation of the straight line passing through the point (0,-5) and parallel to the straight line whose equation is $y = 2x + 3$?

A $x + 2y + 5 = 0$
B $2x - y - 5 = 0$ *
C $2x + 3 = -5$
D $2x - 5y + 3 = 0$
E $2x + y + 5 = 0$

Performance on the trigonometry items was also surprisingly low, even on those dealing with simple applications of elementary formulas as in the case of Item #039. The international mean for this item was 62%, surprisingly low given the nature of the item, but none of the North American groups obtained that high a score. The BC-Grade 12 result, 37%, was very poor although it was higher than either the Ontario-Grade 12 or USA-Pre Calculus results: 23% and 30%, respectively.

Item #039

If cos A = ½, then cos 2A is equal to

A -1/2 *
B 1/2
C -3/2
D 3/2
E 1

The rather dismal state of affairs which appears to exist with respect to geometry, both as regards the content of the curriculum, and students' performance levels, needs to be addressed. A serious effort must be made to improve the present situation in order that teachers of mathematics at all grade levels can do a better job of teaching this important component of the mathematics curriculum.

Elementary Functions and Calculus

This subtest consisted of 46 items of which, as was noted earlier, 27 dealt with calculus. Almost all of the calculus items were inappropriate for British Columbia students, and their performance reflected this fact. The topics dealt with included interpreting and recognizing graphs, trigonometric and logarithmic functions, composition of functions, limits, derivatives, and integrals. The graph in Figure 13 shows how the various groups of students performed on this set of items.

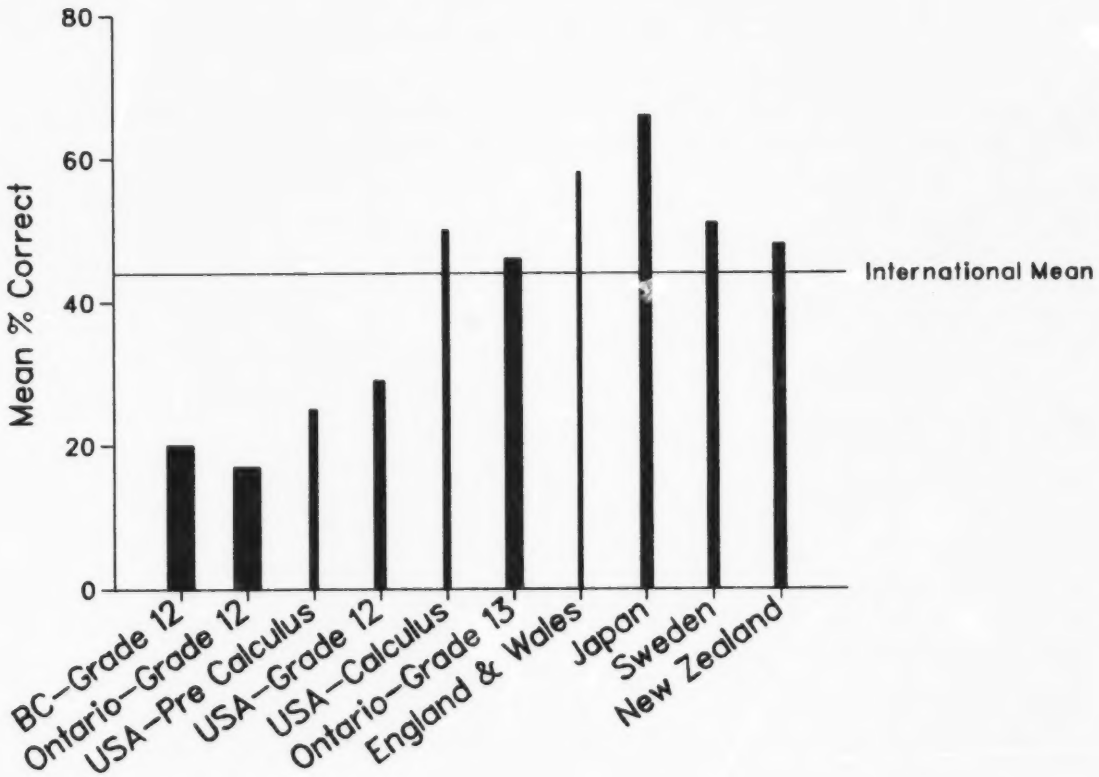


Figure 13. Performance in elementary functions and calculus.

Population B Results

The international mean percent correct on this subtest was 44% and, not surprisingly, the three North American pre-calculus groups did very poorly. BC-Grade 12 and Ontario-Grade 12 scores, at 20% and 17% respectively, were at or below chance level. The performances of the Ontario-Grade 13 and USA-Calculus groups were considerably better.

The item on this subtest which resulted in the best performance by British Columbia students was Item #070, shown below. The BC-Grade 12 *p*-value was 82%, while the Ontario-Grade 12 and USA-Pre Calculus results were 60% and 73%, respectively.

Item #070

If $xy = 1$ and x is greater than 0, which of the following statements is true?

A When x is greater than 1, y is negative.
B When x is greater than 1, y is greater than 1.
C When x is less than 1, y is less than 1.
D As x increases, y increases.
E As x increases, y decreases. *

An item on which British Columbia students' performance was disappointingly low was Item #086.

Item #086

The sum of the infinite geometric series

$1 - 1/2 + 1/4 - 1/8 + \ldots$ is

A 5/8
B 2/3 *
C 3/5
D 3/2
E infinity

Only 39% of BC-Grade 12 students selected the correct response, in spite of the fact that 96% of teachers said they had taught this material. This compares with even poorer performance by Ontario-Grade 12 students whose *p*-value was 8% and USA-Pre Calculus, with 18%. The most frequently selected incorrect response was E: infinity. Almost 25% of British Columbia students chose that answer.

Another rather disappointing result occurred on Item #026.

Item #026

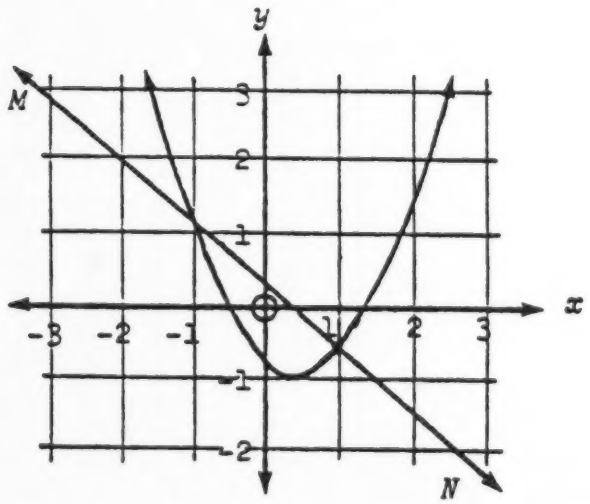
Given $\log_b 2 = 1/3$, $\log_b 32 =$

A 2
B -5
C -3/5
D 5/3 *
E $3/\log_2 32$

The British Columbia OTL result on this item was 98%, indicating that this topic is taught in Algebra 12 classrooms in the province, but the *p*-value was only 48%, much lower than in a number of other countries. Both Ontario-Grade 12 and USA-Pre Calculus had scores of 39%. Such poor results are difficult to understand on an item which is a fairly simple application of a fundamental property of logarithms.

The Elementary Functions and Calculus subtest included a number of items dealing with the use of graphs, and BC-Grade 12 students did quite poorly on all of them. A case in point was Item #054.

Item #054



Population B Results

For what values of x does the function represented by the line $\overleftrightarrow{MN}$ exceed the quadratic function?

A $-1 < x < 1$ *
B $x < -1$ or $x > 1$
C $-3/4 < x < 1\ 1/4$
D $x > 0$
E $x > y$

Only 24% of BC-Grade 12 students selected A as the correct response. Almost twice as many, 40%, chose Response B. It is very disturbing to see such a low level of performance on an item which required only the ability to interpret information presented in a graph. It seems clear that more attention needs to be paid to the study of graphs in the British Columbia mathematics curriculum. In Ontario-Grade 12 and USA-Pre Calculus classes results were marginally better, but still weak: 25% and 28% correct, respectively.

Probability and Statistics

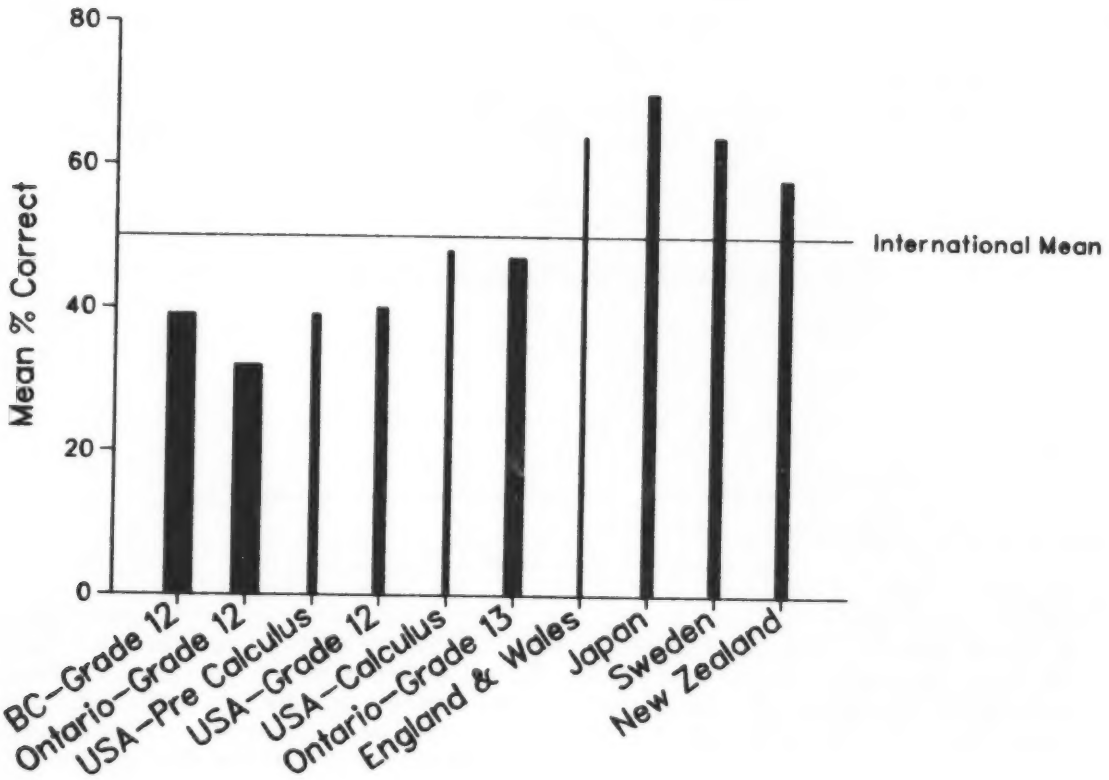


Figure 14. Performance in probability and statistics.

Students in British Columbia did not do very well on these items, but that was not surprising given their lack of Opportunity-to-Learn. On one item, however, students did better than expected; that was Item #030, shown below.

Item #030

A set of 24 cards is numbered with the positive integers from 1 to 24. If the cards are shuffled and if only one is selected at random, what is the probability that the number on the card is divisible by 4 or 6?

A 1/6
B 5/24
C 1/4
D 1/3 *
E 5/12

Only 9% of British Columbia teachers said that their students had been taught the material needed to respond to the item, and the item was also rated inappropriate to the British Columbia curriculum. In spite of this, 58% of BC-Grade 12 students obtained the correct answer; better than the 38% of Ontario-Grade 12 and the 47% of the USA-Pre Calculus groups. As might be expected, students in British Columbia did poorly on other items which dealt with specific concepts, such as the standard deviation of a distribution.

General Comments

At the Population B level, the performance of Algebra 12 students in British Columbia compares favourably with that of students following similar programs in Ontario and the United States. Comparisons with students from other countries are highly problematic given the enormous differences in the curricula offered to students and their much lower rates of retention of students in pre-university mathematics programs.

Certain aspects of the results are rather disturbing. For example, North American students appeared to have great difficulty with items dealing with graphs. This may mean that students are not required to work with graphs sufficiently and, if this is the case, it is indeed unfortunate. Graphs provide a concrete representation of abstract mathematical entities and relationships, and can serve as important aids to understanding. Their use should be emphasized at all grade levels.

Secondly, students generally found items requiring the use of higher order thinking skills difficult. Results on items requiring only the solution of an equation, the recall of a definition or property, and the like were well done; but, when additional steps were required, performance was much lower. This was true, not only in British Columbia, but also in Ontario and the United States. Students need to work with difficult problems, and they need to develop problem-solving skills if they are to be successful in mathematics. Pursuit of that goal must be one of our top priorities.

CHAPTER 4

Conclusion

The Second International Mathematics Study has been an immense undertaking involving over 100 000 students and teachers of mathematics in more than 20 countries. The results and findings will have an important influence on future directions in mathematics education for many years to come. This report, one of the first to be published with comparative data from several countries, is but a first step and much remains to be done by way of understanding the major findings.

One of the major goals underlying British Columbia's participation in the project was a desire to see our curriculum--intended, implemented, and attained--against a backdrop of several others, particularly those from places such as Ontario and the United States, which have comparable social and educational systems. This the project has permitted us to do, and in this report some initial comparisons focussing particularly on students' achievement, have been made.

At the Population A level, Grade 8 students in British Columbia compare very favourably to those in other countries. Overall, their results would place them among the top third of the 20 participating countries, and higher than students from any other English-speaking jurisdiction.

On the other hand, it is clear that students in some other countries, notably Japan, perform much better than our students do and have a very different curriculum. Our Grade 8 mathematics program, like most others at that level in North America, is very general and unfocussed. It includes topics from all the subtest strands included in the study, except, perhaps, descriptive statistics. Moreover, the Grade 8 program recapitulates much of what was done in Grade 7.

The Japanese Population A sample was chosen from the Grade 7 level, and their curriculum at that level includes mostly algebra. They complete the study of arithmetic by the end of Grade 6, and then move to algebra in Grade 7. Our "spiral" curriculum, where we re-visit the same topics several years in succession, may be contrasted with their more "linear" curriculum. Insofar as content is concerned, there would not appear to be any mathematical reason for delaying the introduction of a complete course in algebra or an integrated high school mathematics course until Grade 9. It may be that what we like to think is a spiral curriculum is, in fact, a circular curriculum: we re-visit the same topics several times, but not from a different perspective or in any greater depth.

Conclusion

Geometry is a cause for particular concern at both population levels; indeed, throughout the K-12 mathematics curriculum. We urgently need some specific, well-reasoned suggestions for making sense out of the disarray in which we find ourselves. This is not just a local problem as the discussion in earlier chapters has made clear. The same situation appears to pertain internationally. It is to be hoped that the curriculum revision committees which are presently at work in British Columbia will make suggestions in this regard, and recommend the adoption of appropriate pedagogical materials to improve this state of affairs.

At the Population B level, British Columbia finds itself in a unique position, retaining as it does a larger percentage of the age cohort of students in the last year of secondary school in the study of pre-university mathematics than any other country which participated in the study. This choice leads directly to a number of consequences, and the international study affords an opportunity to examine those consequences and to review the present policy.

For example since there is for all practical purposes only one mathematics course for our Population B students, Algebra 12, and since that course includes almost 40% of Grade 12 students, the content must be restricted. It would not be possible to teach the same course to 40% of an age group, that one might offer to the top 5 or 10%. In other words, there is some question whether or not our best students are disadvantaged by not being permitted to specialize in mathematics at the senior secondary level as they can in other jurisdictions or in other subject areas.

We certainly have some excellent students in Algebra 12. When international comparisons are made among the top 1% and top 5% of students in each country on content which is appropriate to our curriculum, the Algebra subtest for example, our Algebra 12 students do as well as those in almost every other country. The difference is that students in those other countries have been exposed to much more content and in much greater depth than ours have.

To summarize, the results from the achievement testing phase of the Second International Mathematics Study in British Columbia are generally positive, although a number of areas where improvement is needed have been identified. On the whole, the performance of our students, at both levels sampled, compares favourably with those from other jurisdictions on those topics which are appropriate to our curriculum. The results do, on the other hand, raise some important issues about the content of our curriculum and the degree of specialization which should be encouraged. These issues should be addressed as soon as possible.